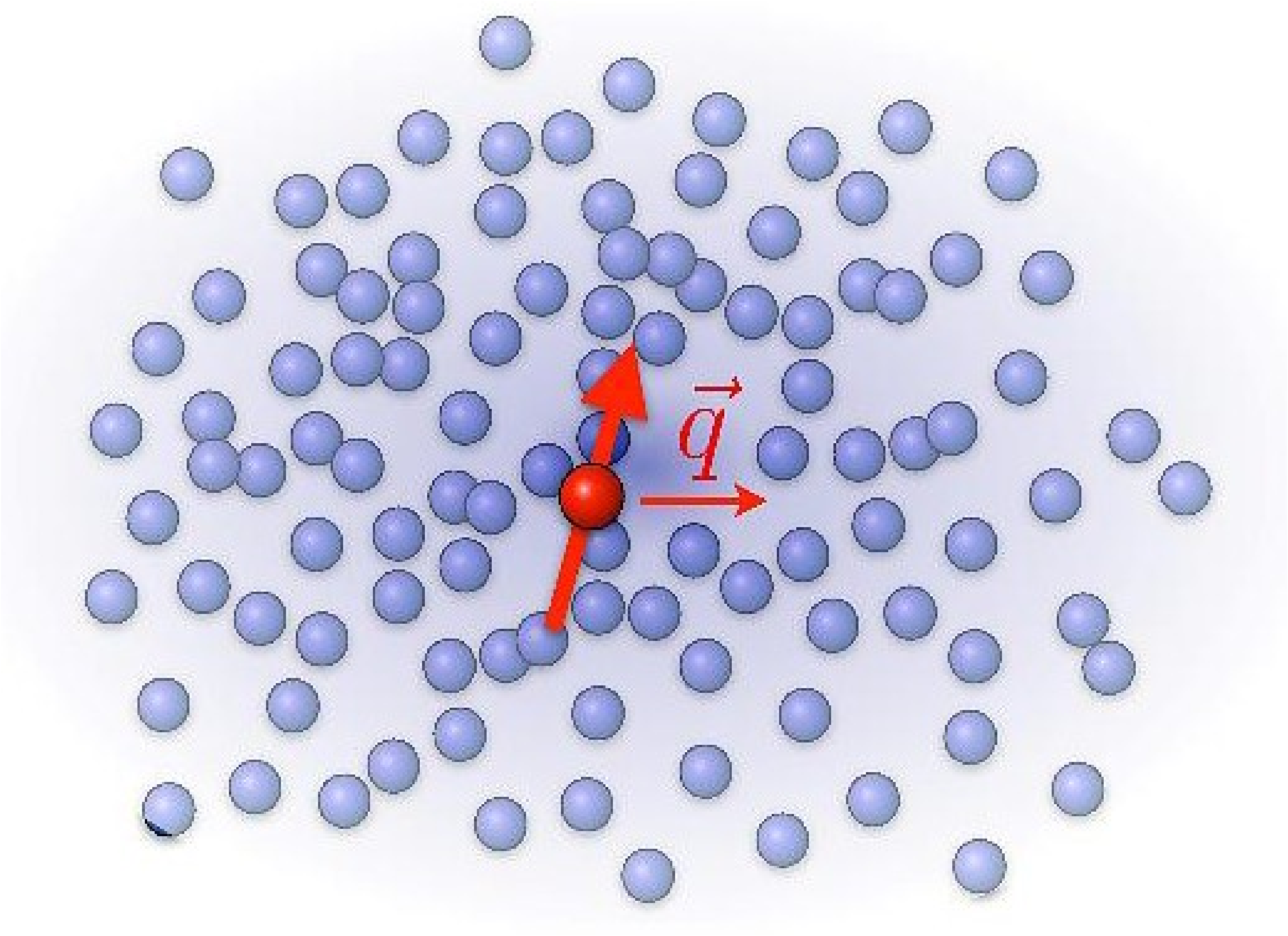
MODELING OF THE IMPURITY DYNAMICS IN ULTRACOLD ATOMIC MEDIA

V.V. Vyborova, Y.E. Shchadilova, A.N. Rubtsov

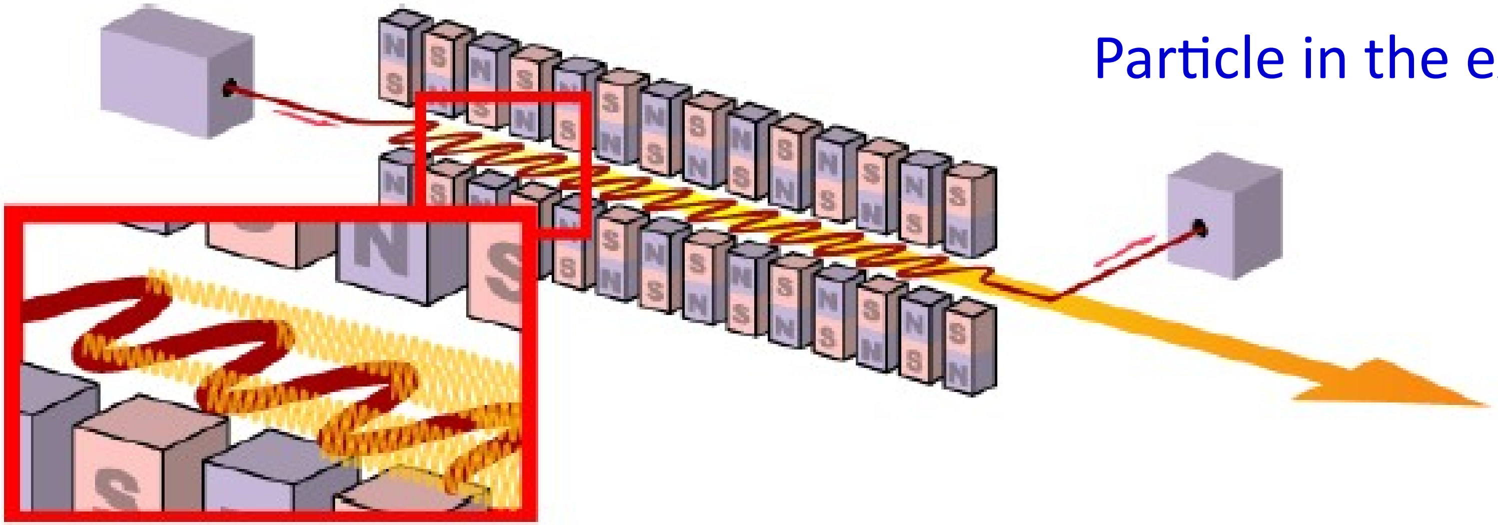
Moscow State University, Russia
Russia Quantum Center, Russia
VNIIA, Russia

AMM-2016, Ekaterinburg

Polaronic problem



$$H = \frac{\hat{P}^2}{2m} + \sum_k V_k e^{ikr}(\hat{a}_k + \hat{a}^\dagger_{-k}) + \sum \varepsilon_k \hat{a}^\dagger_k \hat{a}_k$$



Particle in the external field

Classical Hamiltonian for charged particle interacting with electromagnetic field

$$H = \sqrt{m_e^2 c^4 + c^2(\mathbf{p} - e\mathbf{A}/c)^2}$$

Quantum Hamiltonian in the moving frame

$$H = \frac{\hat{p}^2}{2m} + \sum_k g_k \left(\hat{a}_k e^{i\bar{k}z'} + \hat{a}_k^\dagger e^{-i\bar{k}z'} \right) + \sum_k \bar{\omega}_k \hat{a}_k^\dagger \hat{a}_k$$

$$\bar{\omega}_k = \gamma_0(\omega_k - kv_0) - k_u v_0 \gamma_0$$

$$\bar{k} = \gamma_0(k + k_u - v_0\omega_k/c^2)$$

$$g_k = K\sqrt{2\pi c\hbar e^2/Vk}$$

Consider positive sets $\{p_i\}, \{p'_i\}$. Then

$$\sum_i (\ln p'_i - \ln p_i) p'_i + \left(\ln \sum_i p_i - \ln \sum_i p'_i \right) \sum_i p'_i \geq 0,$$

and the upperbound is reached at $p'_i = C p_i$.
Applied to a stochastic ensemble of imaginary-time trajectories, this reads

$$\langle S' - S \rangle' + \ln \sum_i e^{-S'_i} - \ln \sum_i e^{-S_i} \geq 0.$$

Let us assume Gaussian trial statistics $S' = \bar{a}_i G^{-1}_{ij} a_j$ and optimize G according to the Feynman's principle. It gives:

$$\frac{\delta \langle S \rangle'}{\delta G^{-1}} = -G.$$

Classical Hamiltonian for charged particle interacting with electromagnetic field

$$H = \sqrt{m_e^2 c^4 + c^2(\mathbf{p} - e\mathbf{A}/c)^2}$$

Quantum Hamiltonian in the moving frame

$$H = \frac{\hat{p}^2}{2m} + \sum_k g_k \left(\hat{a}_k e^{i\bar{k}z'} + \hat{a}_k^\dagger e^{-i\bar{k}z'} \right) + \sum_k \bar{\omega}_k \hat{a}_k^\dagger \hat{a}_k$$

$$\bar{\omega}_k = \gamma_0(\omega_k - kv_0) - k_u v_0 \gamma_0$$

$$\bar{k} = \gamma_0(k + k_u - v_0\omega_k/c^2)$$

$$g_k = K\sqrt{2\pi c\hbar e^2/Vk}$$

Consider new variables $a \to a + \alpha \cdot a$ in the path-integral:

$$Z = \int e^{-S[a+\alpha \cdot a, \bar{a}+\bar{a}\cdot\bar{\alpha}]} Da, \bar{a}.$$

Since $Z(\alpha) = const$, and therefore $\frac{\partial \ln Z}{\partial \alpha} = 0$,

$$\left\langle \frac{\partial S}{\partial a_i} a_j \right\rangle = \delta_{ij}.$$

Now, assume Gaussian statistics: $\langle ... \rangle = \frac{\int ... e^{-\bar{a}G^{-1}a} d^n a}{\int e^{-\bar{a}G^{-1}a} d^n a}$.
Integration by parts shows that this condition is equivalent to the Feynman's requirement:

$$\frac{\delta \langle S \rangle'}{\delta G^{-1}} = -G.$$

Small interaction

The difference between $\tilde{G}$ and $\tilde{G}_0$

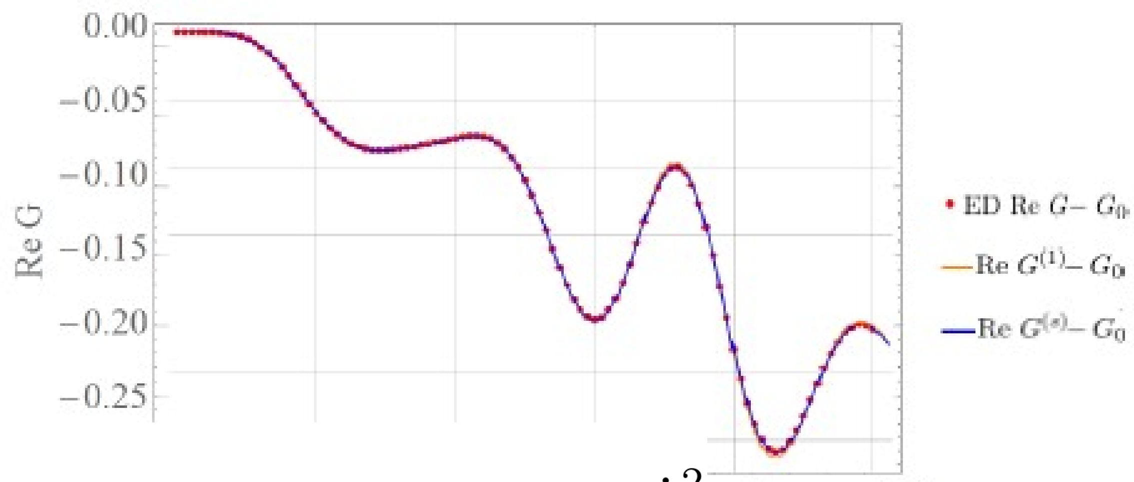
$$\tilde{G}_{t,t'} = \langle (x_t - x_{t'})^2 \rangle \qquad \tilde{G}_0 = \frac{|t-t'|}{\cdots}$$

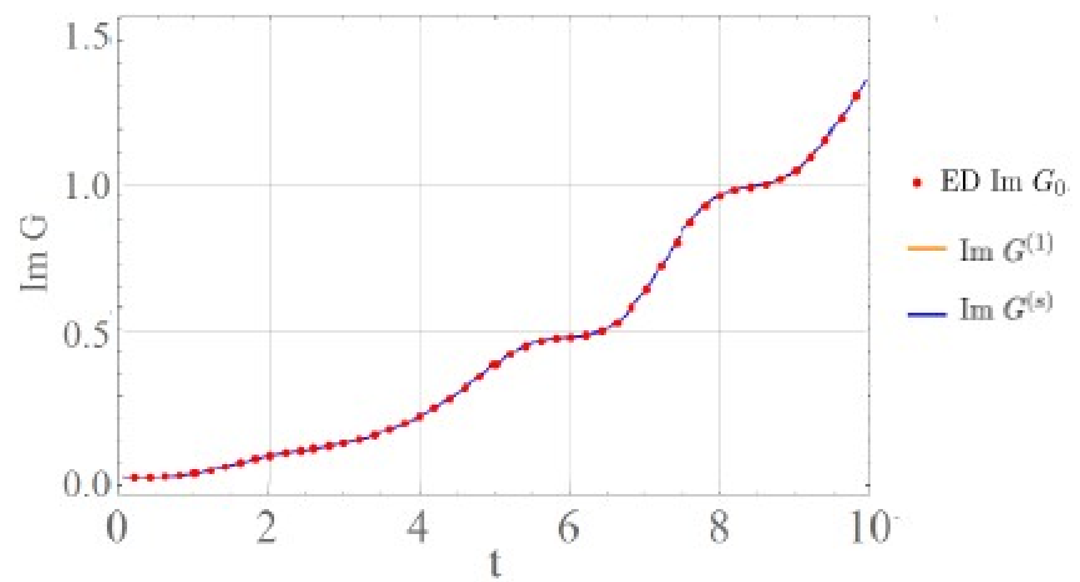
Frölich action

$$S[x] = \frac{m\dot{x}^2}{2}$$

m=1, ω ...

k_1=k_2=0.3, V_k=0.5





Intermediate interaction

Green's function $\tilde{G}$

$$\tilde{G}_{t,t'} = \langle (x_t - x_{t'})^2 \rangle \qquad \tilde{G}_0 = \frac{|t-t'|}{m}$$

m=1, ω_1=3, ω_2=2.1

k_1=k_2=0.5, V_k=1.0